

PROBLEM OF THE MONTH #2

APRIL 2019

Directions: Write a complete solution to the problem below showing all work. Your paper must have your name, W#, and Southeastern email address. Solutions are to be placed in the envelope for Problem #2 located in the Department of Mathematics Office, Fayard 308 by 4:30 p.m., **Thursday, May 9**. No late papers will be accepted.

All papers with a correct solution will be entered in a drawing for a great prize!

Questions concerning the problem of the month should be sent to either Dr. Tilak de Alwis (tdealwis@selu.edu), or Dr. Dennis Merino (dmerino@selu.edu)

Problem: *Minimizing a Distance*

Consider the function $f(x) = \frac{x^3 - 6x^2 + 12x - 6}{x^3 - 6x^2 + 12x - 8}$. The graph of $f(x)$ consists of

two nonoverlapping branches. Find the minimum distance between these two branches. Provide the exact answer.

Hint: First prove, if $P_0(x_0, y_0)$ is a fixed point on one branch of the curve, then for a variable point $Q(x, y)$ on the other branch to have the minimum distance P_0Q , the line segment P_0Q must be perpendicular to the tangent line to the curve at Q . Please refer to the diagram.

