PROBLEM OF THE MONTH #2

NOVEMBER 2020

Directions: Write a complete solution to the problem below showing all work. Your paper must have your name, W#, and Southeastern email address. Solutions are to be sent as a SINGLE PDF FILE to the submission address talwissubmissions@selu.edu, with the subject heading of the email as Problem of the Month #2 – November 2020, by 11:59 p.m., **Monday, November 30**. No late papers will be accepted.

All papers with a correct solution will be entered in a drawing for a great prize!

Questions concerning the problem of the month should be sent to either Dr. Tilak de Alwis (tdealwis@selu.edu), or Dr. Dennis Merino (dmerino@selu.edu)

PROBLEM: *Minimizing an Area*

Consider the parabola given by the equation $y^2 = 4ax$ where "*a*" is a positive real constant. Let O be the origin and $A(k, 0)$ be a fixed point on the x-axis, where $k > 0$. A ***variable line*** ℓ cuts the parabola at two distinct point P and Q as given in the diagram below.

(a) Find the minimum possible area for the triangle OPQ. Be sure to mathematically justify why your answer gives the minimum area. Simplify the answer.

(b) Prove that for variable lines ℓ, the orthocenter of the triangle OPQ always lies on a fixed straight line. Also find the equation of this line. Note that for any triangle, the orthocenter is the point where its three altitudes meet. An altitude of a triangle is the perpendicular line drawn from any vertex to the opposite side.

Note: Partial answers might still be considered. So all submissions are welcome!