**Problem: Triangles and Distances**

Consider the triangle whose vertices are at $(0,0)$, $(2,2)$, and $(-2,2)$. Let $C$ be the set of all points which are inside the triangle with the property that the distance from $(0, 0)$ to a point $(x, y)$ on $C$ is equal to the distance from $(x, y)$ to the line $y = 2$. Find the area of the shaded region $R$ in the diagram below.